Fig. 2. Plot of  $\epsilon F$  versus  $c$  or  $\beta$ .

where

$$F = \frac{\epsilon + 1 - 2c^2}{4c^2} \cdot \frac{s}{(\epsilon - c^2)^{1/2}} \ln \frac{(\epsilon - c^2)^{1/2} + s}{(\epsilon - c^2)^{1/2} - s} - \frac{1 - c}{1 + c} \cdot \left\{ \frac{(\epsilon + 1)(1 - c)(1 + 3c)}{4c^2} \frac{1}{\epsilon^{1/2}} \ln \frac{\epsilon^{1/2} + 1}{\epsilon^{1/2} - 1} + \frac{2(\epsilon - c)}{\Gamma} \ln \frac{\epsilon - c + \Gamma}{\epsilon - 1} \right\} \quad (6)$$

and where  $c = \cos \beta$ ,  $s = \sin \beta$ , and  $\Gamma = [2\epsilon(1 - c) - s^2]^{1/2}$ . For large  $\epsilon$  it is found that

$$F \sim 2(1 - c)^2 / 3\epsilon. \quad (7)$$

Fig. 2 shows the quantity  $\epsilon F$  plotted against  $c$  or  $\beta$  for a range of values of  $\epsilon$ . For  $\beta$  small the radiation is clearly negligible. It rises to a maximum at  $\beta = 180^\circ$ , corresponding to an open-circuit condition; it can be verified that (6) then reduces to (14) in [1] when  $c \rightarrow 1$ .

It can also be shown that as  $\epsilon \rightarrow 1$  (6) takes the limiting form

$$F_{\epsilon=1} = 2 \frac{1 - c}{1 + c} \ln \frac{2}{1 - c} + \frac{s^2}{c^2} \ln s. \quad (8)$$

There may be some coupling between the junction arms, particularly for small  $\beta$ , and this has not been considered in the above calculations. This stipulation apart, it is clear that the spurious radiation can be kept low by using small junction angles and large substrate dielectric constants.

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# Air-Gap Effect in Rectangular Waveguide Containing a Lossy *H*-Plane Dielectric Slab

JOHN B. NESS AND M. W. GUNN

**Abstract**—The propagation coefficient for a partially filled rectangular waveguide containing a lossy *H*-plane slab against the broad wall of the waveguide. The solutions of the dispersion equation show that the attenuation and phase coefficients may be increased as well as decreased by the presence of an air gap. For a fully filled waveguide the effect of an air gap is maximized if the gap is equally distributed at the top and bottom of the sample.

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The authors are with the Department of Electrical Engineering, University of Queensland, St. Lucia, Brisbane 4067, Australia.

#### I. INTRODUCTION

FOR CERTAIN dielectric materials the *H*-plane loaded waveguide provides a convenient method [1] for determining material properties. The theoretical and experimental procedures are simplified if the sample is in complete contact with the broad wall of the waveguide. However, it is often difficult to eliminate air gaps completely due to problems such as imperfections in the waveguide, for example, rounded internal corners or warping of the sample itself. It has been shown previously [2] that the attenuation and phase coefficient decrease rapidly as the sample is moved away from the waveguide

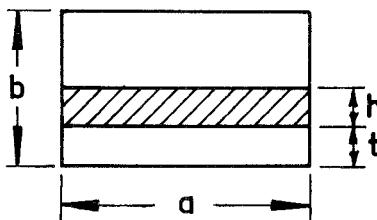


Fig. 1. Partially filled  $H$ -plane loaded waveguide.

wall. However, this is not necessarily the case especially for materials with a significant conduction term such as semiconductors.

## II. ANALYSIS OF THE PARTIALLY FILLED WAVEGUIDE

The propagation coefficients for the LSM modes of the  $H$ -plane loaded waveguide shown in Fig. 1 are obtained from the solutions of the well-known dispersion equation [2]. Typically, the LSM modes will be the dominant modes except when the frequency is well-above cutoff where the  $LS_{01}$  mode, for example, may propagate for samples near the waveguide center [3]. For a sample against the broad wall, it is known [4] that the attenuation coefficient is a nonmonotonic function of sample height and that the phase coefficient may also exhibit nonmonotonic behavior. For semiconductors of low conductivity ( $\sigma \leq 10 \text{ S/m}$ ), the nonmonotonic behavior of the attenuation coefficient can be attributed to a change in propagation from the  $LSM_{11}$  mode to the  $LSM_{12}$  mode. For thick samples ( $h/b > 0.5$ ) the propagation of higher order modes such as the  $LSM_{13}$  mode may cause an additional nonmonotonic response. For semiconductors of higher conductivity ( $\sigma \geq 10 \text{ S/m}$ ), the nonmonotonic response occurs for single  $LSM_{11}$ -mode propagation. It is in the region of the initial nonmonotonic behavior (typically around  $h/b = 0.2$ ) that the air-gap effect is most pronounced.

The effect of the air gap on the attenuation coefficient is shown in Fig. 2 where the attenuation coefficient relative to that for  $t=0$  is plotted as a function of  $t/b$  for various sample thicknesses. The corresponding phase coefficient variation is shown in Fig. 3. For the parameters used, the peak attenuation coefficient occurs for  $h/b = 0.17$ ,  $t=0$ , and this value has been used for the normalizing constant  $\alpha_0$ . The corresponding peak in the phase coefficient occurs at  $h/b = 0.157$ .

For thin samples ( $h/b < 0.1$ ) both the attenuation and phase coefficient decrease as  $t/b$  is increased although the variation in the phase coefficient is only marginal. As the sample thickness is increased the attenuation coefficient decreases more rapidly until for  $h/b = 0.157$  the rate of decrease is a maximum. The phase coefficient, on the other hand, begins to increase such that at  $h/b = 0.157$  the rate of increase is a maximum. The converse situation occurs at  $h/b = 0.17$  where the attenuation coefficient has a maximum rate of increase for small values of  $t/b$  and the phase coefficient has a maximum rate of decrease.

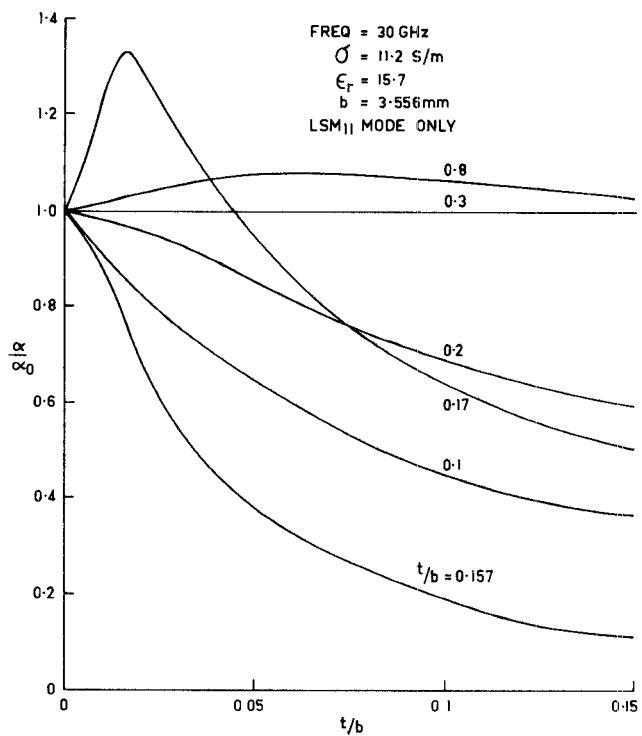


Fig. 2. Relative attenuation coefficient as a function of the air gap for a partially filled waveguide.

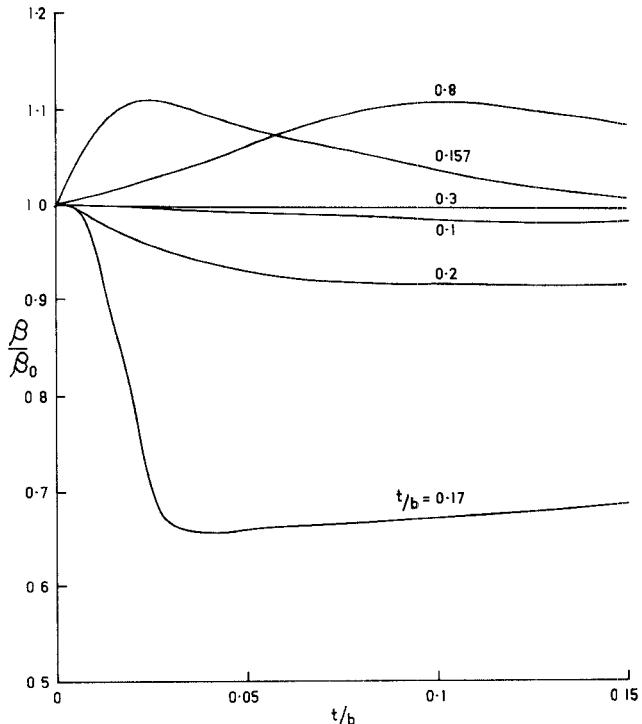


Fig. 3. Relative phase coefficient as a function of the air gap for a partially filled waveguide.

Eventually, for larger values of  $t/b$ , the attenuation coefficient begins to decrease. For sample thicknesses in this region, very small air gaps can cause quite large errors. For example, for  $h/b = 0.17$ , an air gap of approximately 0.05 mm in a  $R$ -band (26.5–40-GHz) waveguide ( $t/b =$

0.015) would cause an error of about +30 percent in the attenuation coefficient and about -20 percent in the phase coefficient.

For thicker samples both the attenuation and phase coefficients decrease relatively slowly with  $t/b$  until for  $h/b=0.3$  the propagation coefficient is virtually independent of  $t/b$ . For very thick samples ( $h/b>0.5$ ), the attenuation and phase coefficients show an increase for small values of  $t/b$ . For lower conductivity samples the effect of the air gap on the propagation coefficient is also quite pronounced in the region of the initial peak in the attenuation coefficient. Where the air gap causes a change in the propagation mode, quite large changes may occur especially in the phase coefficient. The parameters used in Figs. 2 and 3 are such that the  $LSM_{11}$  mode is the dominant propagating mode for the values of  $h/b$  and  $t/b$  shown.

### III. ANALYSIS OF THE FULLY FILLED WAVEGUIDE

The problem of air gaps between the sample and the waveguide when the waveguide is fully filled has been analyzed previously [5], [6]. Usually, the air gap has been accounted for by assuming that it exists along one side and the top, and then corrected values of conductivity and permittivity can be calculated from a perturbation analysis. If the air gaps are assumed to exist only between the sample and the narrow walls of the waveguide, then the propagation coefficients of the  $LSE_{m0}$  modes are found to be virtually independent of the air-gap sizes unless they are quite large. Experimentally, then, these air gaps can usually be made small enough to be ignored. Exact solutions are not obtainable if air gaps exist along both the broad and narrow walls, although both air gaps may occur in the practical case. However, measurements on the fully filled waveguide [7] have shown that for small air gaps, corrections applied for the broad-wall air gap only give results that agree within measurement accuracy. Even small air gaps between the sample and the broad walls of the waveguide can have a marked effect on the propagation coefficient; the distribution as well as the size of the air gaps also is important.

Figs. 4 and 5 show the relative changes in the attenuation and phase coefficients as functions of the air gap for two different distributions. The effect of the air gap is a minimum when the air gap is at the top only and is a maximum when the air gaps at the top and bottom of the sample are equal. For example, for a total air gap of 0.02 mm ( $t/b=0.006$ ) the change in the relative attenuation coefficient is about -4 percent for the equally spaced gap, and there is almost no change for a single air gap. Furthermore, the equally spaced gap also favors the propagation of higher order modes such that even for quite small air gaps ( $t/b\simeq 0.02$ ) the  $LSM_{12}$  mode will propagate with lower loss than the  $LSM_{11}$  (quasi- $TE_{10}$ ) mode. If higher order modes do propagate, then the errors in the attenuation and the phase coefficient may be quite large, of the order of 10 percent or more. Even very small air

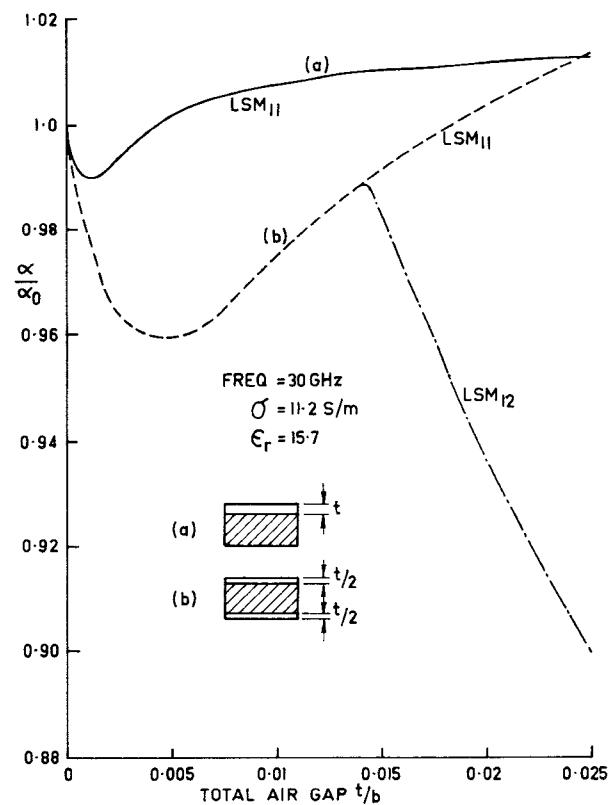


Fig. 4. Relative attenuation coefficient as a function of the total air gap ( $t$ ) for a fully filled waveguide.

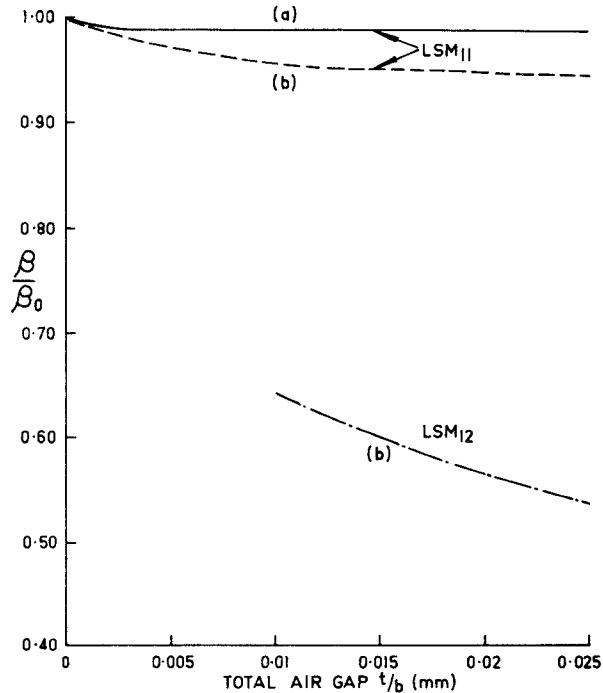


Fig. 5. Relative phase coefficient as a function of the total air gap for a fully filled waveguide.

gaps may cause significant errors if the fully filled waveguide is used to determine material properties. For example, for the parameters shown in Figs. 4 and 5, an air gap of size  $t/b=0.003$  ( $\sim 0.01$  mm) would cause errors in the

measured values of conductivity and permittivity of about 1.5 and 2 percent, respectively. If this air gap were equally distributed at the top and bottom of the sample, the corresponding errors would be about 5 and 3 percent, respectively. Air gaps of this size may easily occur under experimental conditions. Air gaps at the top and bottom of the sample arise from factors such as rounded internal waveguide corners, slight rounding of the sample faces, or an asymmetrical sample cross section.

#### IV. CONCLUSION

For a partial-height *H*-plane sample the presence of an air gap may increase the attenuation or phase coefficient for certain sample thicknesses. This can be compared with previous results [2] which showed for a particular case that the air gap acted to decrease the attenuation and phase coefficients. This increase occurs only for a small range of sample thicknesses, and the maximum rate of increase of the attenuation coefficient is accompanied by the maximum rate of decrease of the phase coefficient (and vice versa). For the fully filled waveguide, the effect of the air gap is maximized when the air gap is equally distributed.

The effect of the air gap also depends, of course, on the conductivity, permittivity, and frequency. For the fully filled waveguide, any air gaps will cause a more rapid decrease in the attenuation coefficient as the conductivity increases. For the partially filled waveguide, on the other hand, lowering the conductivity will tend to increase the effect of the air gap, especially for samples of thickness  $h/b \approx 0.2$  where the air gap may cause propagation of the

higher order modes such as the  $LSM_{12}$  mode. For the partially filled waveguides, these higher order modes will be suppressed as the conductivity increases.

It has been shown that even very small air gaps, which are often difficult to eliminate experimentally, can cause significant changes in the propagation coefficient. Thus at higher frequencies, in particular, it is necessary to eliminate any air gaps or at least estimate the size of the air gap so that the propagation coefficient can be correctly determined.

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